COHERENT MULTICHANNEL RECEPTION OF BINARY MODULATED SIGNALS WITH INDEPENDENT RICIAN FADING

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ABSTRACT

A novel technique to calculate the BER of a q-branch coherent receiver is described. The assumed system consists of an antipodal binary-modulated transmitted signal, with independent Rician fading on each received branch. The method allows simplification of a (q+1)-dimensional integral down to a 1-dimensional integral (16), which is readily amenable to numerical evaluation.

1. INTRODUCTION

The Rician distribution (a generalisation of the Rayleigh distribution) is commonly assumed to be a good candidate for modelling the fast fading in a microwave (e.g. 2GHz) mobile cellular propagation channel for certain deployment scenarios (see for example [1]). The model assumes that the aggregate channel at any instant from mobile transmitter to any basestation receiver antenna element consists of a single 'dominant' component plus a large number of lower-power 'scattered' components. Given suitable spatial or polarisation separation of the receiver antenna elements, we can ensure that their fading is independent. We assume in this paper that the signals from the antenna elements, after suitable downconversion and digitisation, are to be combined using the well-known Maximal Ratio Combiner (MRC) algorithm [2], prior to detection, in order to maximise the resultant signal-to-noise ratio (SNR). We further assume the use of uncoded Binary Phase Shift Keyed (BPSK) or Gray-coded Quaternary Phase Shift Keyed (QPSK) signalling, and require that detection at the basestation of the transmitted data be phase coherent, in order to minimise the resultant bit error rate (BER).

2. THEORETICAL PRELIMINARIES

Suppose that there are q flat-fading (non-dispersive) receiving channels (or 'diversity branches') for coherent reception. We introduce the random variable ρ_i which represents the instantaneous SNR in the i'th diversity branch. We assume Rician fading, and therefore that the random variable ρ_i is a sample from a non-central χ^2 distribution with 2 degrees-of-freedom, and hence has a pdf given by [3 eqn.(2-1-140)]:

$$f(\rho_i)=\begin{cases}\dfrac{1}{2\sigma_i^2}\exp\left(-\dfrac{s_i^2+\rho_i}{2\sigma_i^2}\right)I_0\left[\sqrt{\rho_i}\,\dfrac{s_i}{\sigma_i^2}\right], & \rho_i\ge 0 \\ 0, & \rho_i<0\end{cases} \quad (1)$$

where $I_0(.)$ is the zero-order modified Bessel function of the first kind, and the quantities s_i^2 and $2\sigma_i^2$ represent the mean energy in the dominant and scattered components respectively, as described in [3] (also making the assumption, without loss of generality, that the receiver noise in each diversity branch is independent, with unity variance). We further require the set of ρ_i to be *statistically independent*. The physical interpretation of this is that the aggregate (sum) of the scattered components on each diversity branch, when considered as complex quantities using complex baseband notation [3], must be *statistically uncorrelated* from the aggregate of scattered components on every other diversity branch.

We carry out maximal ratio combination of these multiple diversity branches in order to derive a single detection variable. The 'SNR per bit', ρ, (assuming binary modulation, such that every signal sample carries one bit of information) at the output of the MRC combiner will at any instant be equal to:

$$\rho=\sum_{i=1}^{q}\rho_i \quad (2)$$

consistent with the well-known rule that the SNR at the output of an MRC combiner is equal to the sum of the SNRs at its inputs [2].

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Given an SNR per bit, ρ, at the input to a coherent detector, it can be shown (see [3 eqn. 5-2-5]) that the BER, p, for binary antipodal signaling (i.e. BPSK) is given by:

$$p = Q\left(\sqrt{2\rho}\right) \tag{3}$$

where:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{t^2}{2})\,dt \tag{4}$$

is the well-known expression for the one-sided tail probability of a zero-mean unity-variance Gaussian p.d.f. N(0,1) [3 eqn. 2-1-97]. The same equation for BER also applies for a Gray-coded QPSK (4-PSK) system, if we assume an appropriate definition of ρ such that it continues to represent SNR 'per bit'.

3. ERROR PROBABILITY AT COMBINER OUTPUT

From (1)(2)(3) it can be deduced that the mean BER at the MRC output (averaged over the fading of the combiner output SNR, ρ) is given by:

$$p = \int_0^\infty \int_0^\infty \cdots \int_0^\infty Q\left(\sqrt{2\sum_{i=1}^{q} \rho_i}\right) f(\rho_1) f(\rho_2) \cdots f(\rho_q)\, d\rho_1 d\rho_2 \cdots d\rho_q \tag{5}$$

since, due to their statistical independence, the joint probability of the ρ_i is equal to the product of their marginal probabilities. The problem of finding the BER of our system reduces to the calculation of integral (5) taking into account (1) and (4).

We will make use of the following expression (from [4] No. 8.252-4):

$$\Phi(xy) = 1 - \frac{2x}{\pi} \exp\left(-x^2 y^2\right) \int_0^\infty \frac{\exp\left(-t^2 y^2\right)}{t^2 + x^2}\,dt \tag{6}$$

where $Re(y^2)>0$, and where the function $\Phi(x)$ (from [4] No. 8.252-1, setting y=1) is given by:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\,dt \tag{7}$$

In (5) we use the function $Q(x)$ from (4). Comparing (7) with (4) it is straightforward to show by an appropriate transformation of the variable of integration that:

$$Q(x) = \frac{1}{2}\left[1 - \Phi\left(\frac{x}{\sqrt{2}}\right)\right] \tag{8}$$

Therefore (5) can be written in the form:

$$p = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \frac{1}{2}\left[1 - \Phi\left(\sqrt{\sum_{i=1}^{q} \rho_i}\right)\right] f(\rho_1) f(\rho_2) \cdots f(\rho_q)\, d\rho_1 d\rho_2 \cdots d\rho_q \tag{9}$$

Substituting the following into (6):

$$x = 1; \qquad y = \sqrt{\sum_{i=1}^{q} \rho_i} \tag{10}$$

we obtain the result:

$$\frac{1}{2}\left[1 - \Phi\left(\sqrt{\sum_{i=1}^{q} \rho_i}\right)\right] = \frac{1}{\pi} \exp\left(-\sum_{i=1}^{q} \rho_i\right) \int_0^\infty \frac{\exp\left(-t^2 \sum_{i=1}^{q} \rho_i\right)}{t^2 + 1}\,dt \tag{11}$$

Substituting (11) into (9) and changing the order of integration we obtain the following expression for BER:

$$p = \frac{1}{\pi} \int_0^\infty \frac{1}{t^2+1} \prod_{i=1}^{q} I_i(t)\,dt \tag{12}$$

where:

$$I_i(t) = \int_0^\infty \exp[-\rho_i (t^2+1)] f(\rho_i)\, d\rho_i \tag{13}$$

We will make use of the identity ([4], No. 6.633-4):

$$\int_0^\infty x \exp\left(-\alpha x^2\right) I_0(\beta x)\,dx = \frac{1}{2\alpha} \exp\left(\frac{\beta^2}{4\alpha}\right) \tag{14}$$

Given that the pdf $f(\rho_i)$ has a form of (1), we substitute this into (13), and using (14) we obtain that:

$$I_i(t) = \frac{1}{1 + 2\sigma_i^2 (t^2+1)} \exp\left[-\frac{s_i^2 (t^2+1)}{1 + 2\sigma_i^2 (t^2+1)}\right] \tag{15}$$

Thus the general formula for the calculation of BER, after substituting (15) into (12), will have the form:

$$p = \frac{1}{\pi}\int_0^{\infty}\frac{1}{(t^2+1)}\prod_{i=1}^{q}\frac{\exp\left(-\frac{s_i^2(t^2+1)}{1+2\sigma_i^2(t^2+1)}\right)}{1+2\sigma_i^2(t^2+1)}dt \quad (16)$$

This is our final expression for the BER of the q-branch diversity combiner with independent Rician fading. A multi-dimensional integral (5) has been reduced to a single integral in (16), which is much more amenable to numerical analysis.

4. CONCLUSIONS

In this paper we have described a novel technique to calculate the BER of a q-branch phase-coherent receiver for an antipodal binary-modulated transmitted signal, with independent Rician fading on each received branch. In a related paper [5] by the same authors, we provide more detailed discussion of the propagation scenario, extend the analysis to consider the case of the multi-branch combiner with *dependent* Rician fading at its inputs, and demonstrate the utility of the result to the system designer. Furthermore, we demonstrate in [5] that our key result (16) is consistent with previously-known analytical and simulation results for certain special cases (e.g. the multi-branch Rayleigh and single-branch Rician cases). In closing, we draw the reader's attention to other contemporary work on this topic [6], which includes consideration of more generalised modulation schemes and fading distributions.

5. REFERENCES

[1] J.D.Parsons, *The Mobile Radio Propagation Channel*, Pentech Press. London, 1992

[2] D.G.Brennan, "On the Maximum Signal-to Noise Ratio Realizable from Several Noisy Signals", *Proceedings of the IRE*, vol. 43, October 1955, p. 1530

[3] J.G.Proakis, *Digital Communications*, McGraw-Hill, Third Edition (International Edition), 1995

[4] I.S.Gradshteyn and I.M.Ryzhik I, *Tables of Integrals, Series, and Products*, Academic Press, 1965

[5] D.D.N.Bevan, V.T.Ermolayev and A.G.Flaksman, "Coherent multichannel reception of binary modulated signals with dependent Rician fading", Submitted for publication to *IEE Proceedings-Communications*

[6] M-S.Alouini and A.J.Goldsmith, "A Unified Approach for Calculating Error Rates of Linearly Modulated Signals over Generalized Fading Channels", *IEEE Transactions on Communications*, vol. 47, no. 9, September 1999, pp. 1324-1334