PILOT POWER ALLOCATION FOR CDMA SYSTEMS WITH ANTENNA ARRAYS

John S. Thompson [1] and D. Damian N. Bevan [2]

[1] *Signals and Systems Group, Department of Electrical Engineering, The University of Edinburgh, Edinburgh, EH9 3JL, UK.*
Email: John.Thompson@ee.ed.ac.uk

[2] *Nortel Networks, London Road, Harlow, Essex CM17 9NA, UK.*

ABSTRACT

This paper discusses the uplink of a third generation code division multiple access wireless system using antenna arrays at the base station. Each user sends a pilot signal with their modulated data, whose power can be varied as required by the base station. The uplink is then detected coherently using the pilot signal to estimate the channel coefficients for each mobile. Theoretical results on signal-to-noise ratio (SNR) degradation due to imperfect channel estimation are used to determine the optimum power allocation for the pilot signal in a Rayleigh fading environment. The results can be used for any SNR, for any channel Doppler frequency and for any receiver array size.

I. INTRODUCTION

In code division multiple access (CDMA) mobile communication systems, receiver diversity combining is an important technique to combat the adverse effects of channel fading, which is caused by multipath propagation. Diversity may be obtained through the use of a number of techniques, such as space diversity where two or more widely spaced receive antennas are used. The receiver must then select a set of weights to combine the different antennas' signals and detect the transmitted waveform. If white Gaussian noise corrupts the desired signal, maximal ratio combining maximises the signal-to-noise ratio (SNR) at the combiner output [1].

The weights will vary in time due to variations in the channel and these variations must be tracked at the receiver. In the cdma2000 proposal [2] the mobile will send two different signals using two different orthogonal CDMA spreading codes. The first code channel contains the user's data sequence; the second channel is a pilot signal for channel estimation. A block diagram of such a transmitter is shown in Figure 1. Channel variations mean that only a finite length of the pilot signal can be used for channel estimation at any time. Noise will be present in the estimated combiner weights causing "weight jitter", which deteriorates performance relative to the case where the weights are known exactly.

The question then arises as to what is the optimum ratio of pilot power to signal power, namely the parameter β in Figure 1. This problem has been addressed in [3], which optimised β under the condition that the total transmit power is constant. However, it is the contention of this paper that a more suitable approach is to minimise the total transmitted power while still meeting a fixed bit error ratio (BER) target. Then the receiver must trade off weight error loss (WEL), which is the increase in SNR required to meet a given BER target due to weight jitter, against the additional power transmitted for the pilot signal. Closed form WEL formulae are used here to determine the optimum pilot power allocation for any allowable length of pilot signal used for channel estimation.

This paper is organised as follows: section II summarises the derivation of theoretical results for WEL with antenna arrays. Section III shows how WEL equations may be used to optimise pilot signal power. Section IV provides some example results and finally, section V provides conclusions to the paper.

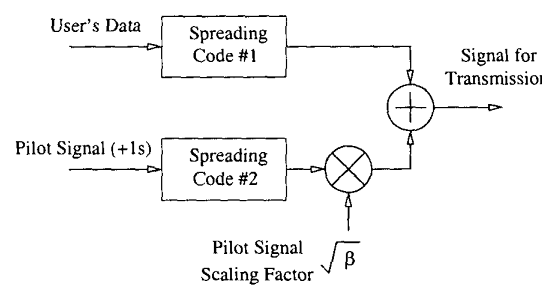

Fig. 1. Conceptual diagram of the mobile transmitter.

II. WEIGHT ERROR LOSS ANALYSIS

This section will discuss two different WEL formulae, one which is exact and the other approximate, which will be used for optimising pilot power. A single transmitter and N antenna receiver system using BPSK modulation is considered. Additive white Gaussian

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noise of equal power is present at each receiver antenna. Each receiver antenna observes an independent, identically distributed Rayleigh fading channel, with mean SNR γ_1. The ratio of pilot-to-signal power is β, so that the SNR of the pilot channel is $\beta\gamma_1$ for one symbol period. Channel effects limit the receiver to integrating pilot signal over D consecutive symbols, so that the final channel estimate has SNR $L\gamma_1$ at each antenna, where $L = \beta D$.

1. Exact WEL Formula: This part of the paper will summarise the derivation of an exact WEL result for BPSK modulation. Appendix C, equation (C-18) of [1] quotes the following bit error ratio (BER) for this configuration, which is called a "clairvoyant channel estimate". The result is:

$$P_e(L,\gamma_1) = \frac{1}{2} - \frac{\mu(L,\gamma_1)}{2}\sum_{k=0}^{N-1}\binom{2k}{k}\left(\frac{1-\mu(L,\gamma_1)^2}{4}\right)^k. \tag{1}$$

The parameter $\mu(L,\gamma_1)$ represents a "cross-correlation coefficient" [1] , and is defined as:

$$\mu(L,\gamma_1) = \frac{\gamma_1}{\sqrt{(1+\gamma_1)(\gamma_1+(1/L))}}.$$

The weight error loss (WEL) can be obtained for any particular L and any required BER as follows; the idea is to find the SNR γ_2 such that $P_e(\infty,\gamma_1) = P_e(L,\gamma_2)$, where γ_1 is the SNR per branch for coherent BPSK. Then the WEL is given by the value γ_2/γ_1.

The value P_e in equation (1) depends *only* on L and γ_1 through the parameter $\mu(L,\gamma_1)$. The function P_e is also a monotonically decreasing function as γ_1 increases in value (and L is fixed) [3]. Thus equality of BER with different values of L indicates that $\mu(\infty,\gamma_1) = \mu(L,\gamma_2)$. Expanding this equality gives:

$$\sqrt{\gamma_1/(1+\gamma_1)} = \frac{\gamma_2}{\sqrt{(1+\gamma_2)(\gamma_2+(1/L))}}.$$

This gives rise to a quadratic equation in γ_2 in terms of γ_1. Now define the total mean SNR of the receiver for both cases as $z_1 = N\gamma_1$ and $z_2 = N\gamma_2$. Solving the quadratic equation for γ_2 gives the WEL as:

$$\begin{aligned}\text{WEL} &= \left(\frac{z_2}{z_1}\right) = \left(\frac{\gamma_2}{\gamma_1}\right) = \frac{1}{2}(1+(1/L)) \\ &\pm\left(\frac{\sqrt{(1+(1/L))^2+(4N/(z_1L))}}{2}\right). \end{aligned}\tag{2}$$

It is apparent that $z_2 \geq z_1$ must hold, so the $\pm$ sign in equation (2) becomes just $+$. So far, only BPSK modulation has been considered. However, equation (C-16) of Appendix C in [1] shows that the symbol error ratio (SER) for any M-PSK modulation scheme depends only on the receiver SNR through the parameter μ. Therefore, the WEL equation applies unaltered to the SER for any M-PSK modulation scheme.

2. Approximate WEL Formula: Equation (33) of [4] provides the equivalent Bhattacharyya bound that corresponds to equation (1):

$$P_e(L,\gamma_1) = \left(1+\frac{\gamma_1}{(1/L\gamma_1)+(1/L)+1}\right)^{-N}. \tag{3}$$

Again, making the substitution $P_e(\infty,\gamma_1) = P_e(L,\gamma_2)$ gives the WEL as:

$$\text{WEL} = \left(\frac{z_2}{z_1}\right) = 1 + \frac{1}{L} + \frac{N}{Lz_1}. \tag{4}$$

This approximation is clearly simpler to manipulate than the exact result of (2). As $z_1 \to \infty$, both WEL formulae tend towards $1+(1/L)$, the "high SNR" result obtained in [5].

III. Optimising Transmit Power Allocation

In this section, an application of the WEL result to selecting the optimum pilot power ratio β is presented. The idea is similar to that discussed in [6], namely to determine the value of β that minimises the total transmitted power. This consideration is important in CDMA systems, where reducing the transmit power will increase system capacity. In the cdma2000 proposal [2], the power of the pilot signal used by the mobile may be adapted by the base station depending on the number of receiver antenna elements, required SNR and so on.

The SNR parameter z_2 does not include the transmit power required for the pilot signal. The SNR parameter z_3 *includes* the pilot power overhead and is defined as:

$$z_3 = z_2(1+\beta). \tag{5}$$

An expression for (z_3/z_1) may be found by substituting for z_3 using equation (5) and then inserting the WEL formula in place of the ratio (z_2/z_1), either (2) or (4). It is now possible to define a function $f = (z_3/z_1)$ that contains both the WEL contribution and the overhead due to the transmission of a pilot signal. This function may be investigated to find the value β_0 which minimises f and thus provides the best compromise of pilot signal strength and channel estimation error.

This minimisation can be performed analytically to find a closed form solution for β_0. This is achieved by differentiating the function $f = (z_3/z_1)$ with respect to the parameter β. Setting this derivative to zero allows the value β_0 that minimises $f(\beta)$ to be found. This procedure will now be summarised for both the WEL solutions described above.

1. Exact WEL Formula: The function f may be obtained by substituting (5) into (2), giving:

$$f(\beta) = \left(\frac{z_3}{z_1}\right) = \frac{(1+\beta)}{2}\left(1+\frac{1}{\beta D} + \sqrt{\left(1+\frac{1}{\beta D}\right)^2 + \frac{4N}{z_1\beta D}}\right). \quad (6)$$

Differentiating f then gives:

$$\frac{\partial f}{\partial \beta} = \frac{1}{2}\left(1+\frac{1}{\beta D}+\sqrt{\left(1+\frac{1}{\beta D}\right)^2+\frac{4N}{z_1\beta D}}\right) - (1+\beta) \cdot \left(\frac{1}{2\beta^2 D} + \frac{2(1+(1/(\beta D)))/(\beta^2 D) + (4N/(z_1\beta^2 D))}{4((1+(1/(\beta D)))^2+(4N/(z_1\beta D)))^{\frac{1}{2}}}\right).$$

Equating this function to zero allows the result to be simplified into a quadratic equation in β:

$$(D^2 z_1 - D z_1 - ND)\beta^2 + (2ND)\beta + (z_1 - ND - Dz_1) = 0.$$

The solutions for β_0 are then given by:

$$\beta_0 = \frac{ND \pm \sqrt{z_1(z_1+N)D(D-1)^2}}{D(N - z_1(D-1))}. \quad (7)$$

Numerical experience indicates that usually only one of these solutions is a valid solution to $\partial f/\partial\beta = 0$. This outcome arises because a squaring operation is used to remove the square root terms in $\partial f/\partial \beta$ when simplifying to the quadratic equation. It is the squaring operation which appears to introduce an extra, possibly incorrect solution for β_0. The correct solution may be verified as follows: (1) ignoring negative values of β_0, (2) checking that $\partial f/\partial \beta = 0$ or (3) picking value of β_0 which has the smaller value of $f(\beta_0)$. In the results obtained for this paper, it was found that substituting "$-$" for "$\pm$" into (7) always gave the correct solution, though it has not been proven that this is the case for all possible scenarios.

2. Approximate WEL Formula: This time, the function f is obtained by substituting (5) into (4), giving:

$$f(\beta) = \left(\frac{z_3}{z_1}\right) = (1+\beta)\left(1+\frac{1}{\beta D}+\frac{N}{\beta D z_1}\right). \quad (8)$$

Differentiating f is somewhat simpler in this case, giving:

$$\frac{\partial f}{\partial \beta} = 1 - \frac{1}{\beta^2 D} - \frac{N}{\beta^2 D z_1}.$$

Setting $\partial f/\partial\beta = 0$ gives a single solution for β_0:

$$\beta_0 = \sqrt{\frac{1}{D}\left(1+\frac{N}{z_1}\right)}. \quad (9)$$

Essentially the same result has been obtained in [7], through direct analysis of estimation error.

The behaviour of the parameter β_0 and the ratio (z_3/z_1) will now be discussed for different scenarios.

A. Pilot Power Ratio Trends

Under conditions of high SNR, i.e. $z_1 \to \infty$, $N/z_1 \to 0$. In this situation, both solutions for β_0 simplify to:

$$\beta_0 = \sqrt{\frac{1}{D}}. \quad (10)$$

Substituting this back into either equation for $f(\beta_0)$ gives:

$$f(\beta_0) \approx (1+\beta_0)\left(1+\frac{1}{\beta_0 D}\right) = \left(1+\sqrt{\frac{1}{D}}\right)^2. \quad (11)$$

It is apparent that for high SNRs, the larger the value of D, the smaller is the pilot power ratio and the resulting performance loss due to channel estimation. As might be expected, the use of pilot signals is more efficient for low Doppler rate channels than for very fast fading environments. In the extreme case where $D = 1$, β_0 becomes 1 which is consistent with [6]. This gives a 6 dB increase in SNR, half of which (3 dB) is due to pilot overhead, the other 3 dB is attributable to channel estimation errors. However, the pilot overhead may be eliminated by using differential detection instead [1]. At lower SNRs, the exact WEL result always has $\beta_0 = 1$ when $D = 1$, but the approximate WEL formula gives $\beta_0 = (1 + (N/z_1))^{1/2} > 1$.

The SNR per antenna γ_1 may become small for two reasons, either N becomes large or the total SNR z_1 also becomes small. For the exact WEL result, the parameter β_0 tends towards 1 as $\gamma_1 \to 0$. However, the approximate WEL result gives a value $\beta_0 \to \infty$ as $\gamma_1 \to 0$. Thus the approximate WEL result again over-estimates the required pilot power allocation.

B. Time-Division Multiplexed Pilot Symbols

Before moving on to present some example results, it should be noted that the analysis for the pilot signal scheme can be reformulated for time-division multiplexed (TDM) reference signals [4]. In this situation, the channel reference is provided by L known pilot symbols which are time multiplexed with the data stream at the transmitter. Clearly, the analysis of this paper may be used to consider how L can be optimised for such a case. Indeed the analysis of [3] has been modified in [8] to determine the optimum number of pilot symbols, again under a constraint that the total transmitter power is fixed.

IV. RESULTS AND EXAMPLES

Initially, an $N = 2$ antenna diversity receiver is considered: for ideal coherent BPSK, the required SNR z_1 for a mean BER of 1% is 8.45 dB [1]. Given this value, the parameter z_3 is then calculated using the exact analysis of equations (2) and (5). Figure 2 plots the ideal BPSK result along with equivalent results for z_3 for three different values of D. The task is then to find the optimum value of β_0 corresponding to the minima of these curves. Table I presents results for β_0 and z_3 obtained using both the exact and approximate WEL formulae. It can also be seen that when the channel fading is rapid, and hence the pilot integration time D is short, the unavoidable performance loss becomes higher.

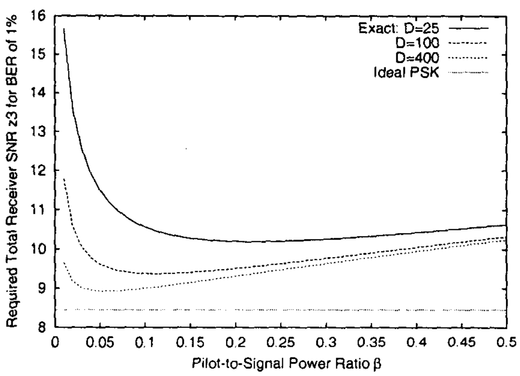

Fig. 2. Plot of the total required SNR value z_3 for a BER of 1% vs the pilot power ratio β, with $N = 2$ receiver antennas.

D	Exact WEL		Approx WEL	
	β_0	z_3	β_0	z_3
25	0.217	10.19 dB	0.227	10.23 dB
100	0.111	9.38 dB	0.113	9.39 dB
400	0.0560	8.92 dB	0.0567	8.93 dB

TABLE I

TABLE OF RESULTS FOR β_0 AND z_3, OBTAINED FROM THE EXACT AND APPROXIMATE WEL FORMULAE, FOR A SCENARIO WITH $N = 2$ ANTENNAS AND A BER TARGET OF 1%.

Figure 3 provides results for β_0 and $f(\beta_0)$, using equations (7) and (6) respectively, for the exact WEL formula. The curves are plotted against integration period D for different array sizes N. In all cases, the SNR parameter z_1 is assumed to be 10 dB. Part (a) shows results for β_0 which are generally increasing as the array size increases. The corresponding results in part (b) show that the function $f(\beta_0)$ also increases with N. This may be attributed to the fact that increasing N reduces the received SNR per antenna γ_1. This in turn increases the level of noise present on the channel estimate for each antenna. This causes the optimum value of β_0 to increase, in an attempt to obtain improved channel estimates. The overall result is that the combined overhead due to pilot signals and combining losses increases significantly as N becomes large. Finally, it should be noted that the high SNR curve lies reasonably close to results for small N in Figure 3 (a) and (b), but increasing N gives significantly larger values of β_0 and $f(\beta_0)$ because the SNR per antenna γ_1 is reducing.

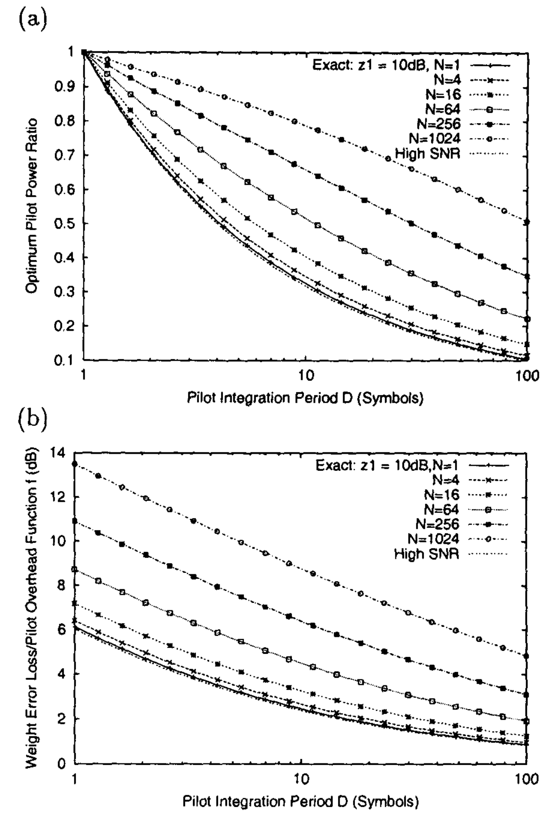

Fig. 3. Plot of (a) the optimum pilot power ratio β_0 and (b) the corresponding performance loss function $f(\beta_0)$ vs the integration period D. The SNR parameter $z_1 = 10$ dB and the array size N is varied from 1-1024. For comparison, high SNR curves for equations (10) and (11) are also shown.

Figure 4 presents comparisons of the formulae obtained from both the exact and approximate WEL formulae, i.e. equations (6), (7) and (8), (9) respectively. In this scenario, the integration period D is set to 100 symbols. Part (a) shows results for the parameter β_0, which are generally increasing as N increases. In addition, reducing the SNR parameter z_1 from 12 dB to 0 dB also further increases the value of β_0. It may

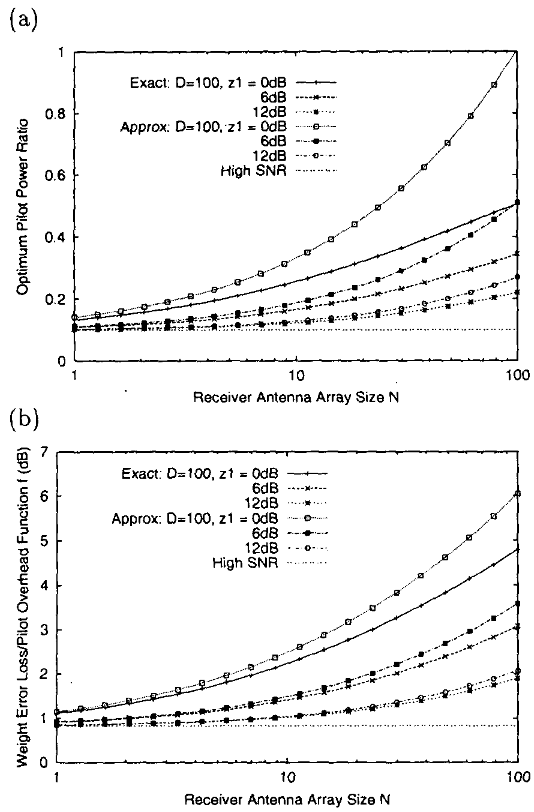

Fig. 4. Plot of (a) the optimum pilot power ratio β_0 and (b) the corresponding performance loss function $f(\beta_0)$ vs the array size N. The integration period $D = 100$ symbols and the SNR parameter z_1 varies from 0 to 12 dB. For comparison, high SNR curves for equations (10) and (11) are also shown.

be seen that for large values of N or small SNR values, the "approximate" curves tend to over-estimate the value of β_0, when compared with the equivalent "exact" curves. This is because the approximate result for WEL in equation (4) contains a term proportional to N/z_1, whereas for the exact result the WEL is roughly proportional to $\sqrt{N/z_1}$. Therefore, as z_1 is reduced, the approximate WEL increases more rapidly than the exact result. However, when N is small or z_1 is large, both sets of curves provide good agreement to the high SNR result. Part (b) provides results for $f(\beta_0)$ and these show similar trends to part (a). For large values of z_1 and small values of N, the two sets of curves tend to coincide with the high SNR result. As z_1 is reduced or N is increased, the approximate result yields higher values of $f(\beta_0)$ than the true value.

V. Discussion and Conclusions

This paper has presented and compared two theoretical results for optimising pilot power allocation in the uplink of a cdma2000 system. An exact result has been obtained from the analysis of BER formulae in [1], but the formula is complex to manipulate and applies only to the case of equal power antenna diversity branches. An approximate formula has also been obtained from the BER analysis of [4]. These approximate results are easier to manipulate, but tend to overestimate the pilot power ratio β_0 and the loss function $f(\beta_0)$. However, the approximate analysis can also be extended to channels with unequal power branches, etc., by direct calculation of the channel estimate mean squared error following [7]. Both the exact and approximate WEL results simplify to the same "high SNR" WEL formula. The β_0 results for the "high SNR" case can be used as an initial estimate. This can be subsequently refined using either one of the sets of formulae presented in this paper.

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