Channel Estimation Errors And Pilot Power Allocation For Wireless Systems With Antenna Arrays

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Abstract. This paper discusses wireless systems using antenna arrays at both the transmitter and receiver. Space-time coding is used to exploit the parallelism of the radio channels formed. Theoretical results on signal-to-noise ratio (SNR) degradation due to imperfect channel estimation are used to determine the optimum ratio of pilot to data symbols to be used in a Rayleigh fading environment. Exact results initially are obtained for the single transmit antenna case. These results are then extended to multiple transmit antennas, using analysis of an upper bound on error probability. The results can be used for any SNR, for any channel Doppler frequency and for any transmitter and receiver array size.

1 INTRODUCTION

In wireless communication systems, space diversity combining at the receiver is a well-known technique to combat the adverse effects of Rayleigh fading, which is caused by multipath propagation [1]. More recently, there has been interest in the use of multiple antennas at both transmitter and receiver. For example, space-time codes [2] have been designed specifically for this case, to exploit fully the diversity present in such multiple-input multiple-output (MIMO) channels. A diagram of such a MIMO system is shown in Figure 1.

A practical system must obtain accurate channel estimates in order to provide good detection performance at the receiver. Channel variations over time mean that only a finite number of pilot or training signals can be usefully used for channel estimation at any time. Hence, noise will be present in the estimated combiner weights which deteriorates performance relative to the case where the weights are known exactly. In most wireless systems, the receiver must attain a target bit error ratio (BER) to provide adequate performance. Then, a parameter called the weight error loss (WEL) quantifies the increase in SNR required to meet a given BER target, taking into account losses arising from noisy channel estimates. The use of pilot signals consumes transmit power, which cannot be devoted to the message that is being sent. The WEL can thus be traded off against the loss in power efficiency that arises from pilot signals.

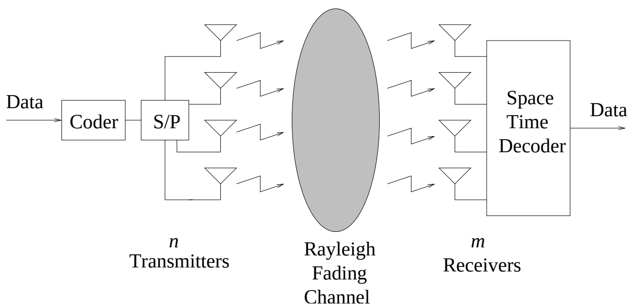

Figure 1: A general communications system employing n transmit and m receive antennas, using space-time trellis coding.

This problem has been addressed in [3] for the case of a single transmit antenna, where the pilot power is optimised under the condition that the total transmit power is constant. However, this paper will investigate what happens when the receiver uses a more realistic constraint, which is to minimise the mobile's total transmit power, while still meeting a given bit error ratio (BER) target. The remainder of the paper is organised as follows: section II provides the channel model to be used throughout the paper. Section III investigates the case where a single transmit antenna is used; section IV then provides general analysis of MIMO-based systems. Section V presents numerical results to highlight some of the major points made during the analysis. Finally, section VI presents the conclusions of the paper.

2 BASIC CHANNEL MODEL

This section will define a generic channel model, very similar to that described in [2], to be used throughout the rest of the paper. A single user system using n transmit antennas and m receive antennas is considered, which will be denoted as an (n, m) system. The transmitter uses a space-time coder to encode a data stream to yield n output data streams. The coded symbol for ith antenna is denoted as c_l^i with l being the symbol number. The space-time symbol $\mathbf{c}(l) = [c_l^1, c_l^2, \ldots, c_l^n]$ is the vector containing the lth group of coded symbols for simultaneous transmission.

It is assumed that one frame of data consists of both pilot and data space-time symbols. The pilot data is sent first and it is assumed that L such space-time symbols are devoted to channel estimation. The n pilot sequences sent by the n transmit antennas are assumed to be mutually orthogonal, in order that they can be separately identified at each receive antenna. The number of encoded space-time symbols generated by the space-time coder for transmission in one frame is D space-time symbols. The pilot-data symbol ratio $\rho = L/D$ is an important quantity for optimisation, a topic which is addressed later in the paper. A diagram of this frame structure of space-time symbols is shown in Figure 2.

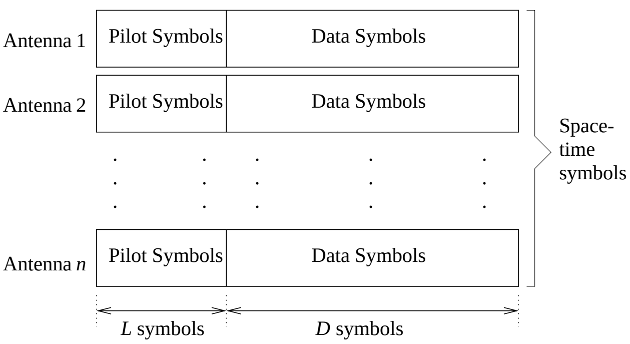

Figure 2: A diagram showing the layout of pilot and data space-time symbols in one frame.

The following assumptions will now be made to model the radio channel:

(a1) The nm channels formed between the transmit and receive antennas are statistically independent of each other and are frequency non-selective. The channel is fading sufficiently slowly such that each channel coefficient remains fixed during one frame of symbols.

(a2) The channel coefficients $\{\alpha_{i,j}, i = 1 \ldots n, j = 1 \ldots m\}$ are zero mean, complex Gaussian random variables, with mean power 1.

(a3) Each receive antenna's signal is corrupted by complex, additive white Gaussian noise of zero mean and finite variance.

(a4) The coded, modulated data symbols c_l^i are assumed to be M-PSK modulated with a constellation radius of unity.

Using these assumptions, the receiver model will now be introduced.

The jth receive antenna demodulates the received signal to baseband. The resulting waveform is matched filtered with the transmitter pulse shape and the output sampled at the symbol rate. The sampled signal for the jth antenna and lth transmitted symbol may be written as:

$$r_l^j = \sqrt{E_s} \sum_{i=1}^{n} \alpha_{i,j} c_l^i + \eta_l^j, \qquad (1)$$

where $\sqrt{E_s}$ is a transmitter scaling factor. The scalar η_l^j represents additive white Gaussian noise. The noise has double-sided power spectral density $N_0/2$ W/Hz and the variance of η_l^j is assumed to equal N_0. The SNR of each symbol c_l^i at the jth antenna, when averaged over Rayleigh fading, can be seen to be E_s/N_0 (ignoring the co-channel interference from the other antennas).

The jth receive antenna can average the received baseband signal for the first L symbols to estimate the channel coefficients $\alpha_{i,j}$. Since the pilot signals from all transmit antennas are mutually orthogonal, the resulting channel estimate for $\alpha_{i,j}$, denoted as $\beta_{i,j}$, may be written as:

$$\beta_{i,j} = \frac{1}{\sqrt{E_s} L} \sum_{l=1}^{L} r_l^j c_l^i = \alpha_{i,j} + \epsilon_{i,j}. \qquad (2)$$

The quantity $\epsilon_{i,j}$ represents estimation error due to the residual level of noise present in the channel estimate. Equations (1) and (2) show that $\epsilon_{i,j}$ is a zero mean, complex Gaussian random variable with variance $\sigma_e^2 = N_0/(E_s L)$. An equivalent method of sending pilot signals in code division multiple access (CDMA) systems is described in [4].

Now that the channel and receiver models have been defined, the next two sections will consider weight error loss and pilot optimisation issues for two cases. Initially, the case of a single transmit antenna ($n = 1$) will be considered. Secondly, the analysis of multiple transmit antenna systems employing space-time coding will be discussed.

3 SINGLE TX ANTENNA CASE

This section considers the $(1, m)$ case and is split into two parts. The first part considers the WEL due to noisy channel estimates, ignoring the pilot overhead. The second part considers the optimisation of the value of the pilot-data symbol ratio ρ.

3.1 WEL FORMULA

This part of the paper will present an exact WEL result for the bit error ratio (BER) for uncoded BPSK modulation. Appendix C, equation (C-18) of [1] quotes the following BER expression for coherent BPSK in Rayleigh fading for a $(1, m)$ system, which uses maximal ratio combining. Since the pilot overhead is ignored, this result refers to a "clairvoyant channel estimate". The result is:

$$P_e(L, E_s/N_0) = \frac{1}{2} - \frac{\zeta(L, E_s/N_0)}{2} \sum_{k=0}^{m-1} \binom{2k}{k} \left(\frac{1 - \zeta(L, E_s/N_0)^2}{4} \right)^k \quad (3)$$

The parameter $\zeta(L, E_s/N_0)$ represents a "cross-correlation coefficient" [1, Table C-1] , and is defined as:

$$\zeta(L, E_s/N_0) = \frac{E_s/N_0}{\sqrt{(1 + E_s/N_0)(E_s/N_0 + (1/L))}}.$$

The WEL can be obtained for any particular value of L and any required BER as follows; the idea is to find the WEL value such that:

$$P_e(\infty, E_s/N_0) = P_e(L, \mathrm{WEL} \cdot E_s/N_0),$$

where E_s/N_0 is the required SNR per receive antenna for ideal coherent BPSK.

The value P_e in equation (3) depends *only* on L and E_s/N_0 through the parameter $\zeta(L, E_s/N_0)$. The function P_e is also a monotonically decreasing function as E_s/N_0 increases in value (and L is fixed) [3]. Thus equality of BER with different values of L indicates that $\zeta(\infty, E_s/N_0) = \zeta(L, \mathrm{WEL} \cdot E_s/N_0)$. Expanding this equality gives:

$$\sqrt{(E_s/N_0)/(1 + (E_s/N_0))} = \frac{\mathrm{WEL} \cdot E_s/N_0}{\sqrt{(1 + \mathrm{WEL} \cdot E_s/N_0)(\mathrm{WEL} \cdot E_s/N_0 + (1/L))}}.$$

This gives rise to a quadratic equation for WEL in terms of E_s/N_0. Solving the quadratic equation gives the WEL as:

$$\mathrm{WEL} = \frac{1}{2}(1 + (1/L)) \pm \left(\frac{\sqrt{(1 + (1/L))^2 + (4/(L \cdot E_s/N_0))}}{2} \right) \quad (4)$$

It is apparent that WEL must be larger than 1, so the $\pm$ sign in equation (4) becomes just $+$. The total received energy across the m antennas for each *information bit* may be written generally as $E_b = mE_s/b$, where b is the number of information bits represented by one modulated symbol (in the particular case here of uncoded BPSK, $b = 1$). So (4) may also be expressed as:

$$\mathrm{WEL} = \left(1 + \frac{1}{L}\right) + \left(\sqrt{\left(\frac{1 + (1/L)}{2}\right)^2 + \frac{m}{bL \cdot E_b/N_0}} - \frac{(1 + (1/L))}{2} \right) \quad (5)$$

So far, only BPSK modulation has been considered. However, equation (C-16) of Appendix C in [1] shows that the symbol error ratio (SER) for any M-PSK modulation scheme depends only on the receiver SNR through the parameter ζ. Therefore, the WEL equation applies unaltered to the raw SER for any M-PSK modulation scheme.

When the E_b/N_0 value is large, i.e. $N_0/E_b \to 0$, the WEL formula (5) simplifies to:

$$\mathrm{WEL} = 1 + \frac{1}{L}. \quad (6)$$

This is the same as the "high SNR" result obtained in [5]. It suggests that WEL at high SNR, for a given number of pilot signals, is independent both of the number of receive antennas and of the order of the M-PSK constellation. However, if the E_s/N_0 value to achieve a target BER is small due to large array sizes m, for example, the WEL formula will deviate significantly from the high SNR result. Indeed, if L and E_b/N_0 are constant and $m \to \infty$, the WEL becomes unbounded in value.

The same behaviour has been observed in single antenna spread spectrum channels [6]. As the bandwidth of the system tends to infinity, it is able to resolve infinitely many paths. Unfortunately, the channel's mutual information then goes to zero, because the receiver is unable to identify the infinite number of channel path amplitudes adequately. The *critical parameter* $m_{crit} = (bLE_b/N_0)$ has been identified in [7] as the number of antennas for which the WEL begins to deteriorate from the high SNR approximation. It is the value of m for which the right hand term in the square root part of equation (5) is 1. Clearly, when $m \gg m_{crit}$, the second bracketed term on the right hand side of equation (5) will become non-negligible. Then, the deviation of WEL from high SNR conditions will be significant.

3.2 OPTIMISING THE PILOT-DATA SYMBOL RATIO

In this subsection, an application of the WEL result to selecting the optimum pilot-data symbol ratio ρ is presented. The idea is similar to that discussed in [8], namely to determine the value of ρ that minimises the total transmit power per information symbol, including the overhead due to the transmission of pilot signals. The chosen value of ρ will minimise the joint contributions of WEL and pilot overhead to the total transmit power.

The WEL result in equation (4) does not include the transmit power required for the pilot signal. A new function $F(\rho)$ can be defined, which includes the overhead due to WEL and to the pilot signal. It may be written as:

$$F(\rho) = \mathrm{WEL} \cdot (1+\rho) = \frac{(1+\rho)}{2}\left(1+\frac{1}{\rho D}+ \sqrt{\left(1+\frac{1}{\rho D}\right)^2+\frac{4}{\rho D E_s/N_0}}\right). \quad (7)$$

The substitution $L = \rho D$ has been made in(4) to obtain this result. This function may be investigated to find the value ρ_0 which minimises F and thus provides the best compromise of pilot SNR and channel estimation error. This minimisation can be performed analytically to find a closed form solution for ρ_0. This is achieved by differentiating the function $F(\rho)$ with respect to the parameter ρ. Setting this derivative to zero allows the value ρ_0 that minimises $F(\rho)$ to be found. Differentiating $F(\rho)$ gives:

$$\frac{\partial F}{\partial \rho} = \frac{1}{2}\left(1+\frac{1}{\rho D}+\sqrt{\left(1+\frac{1}{\rho D}\right)^2+\frac{4}{\rho D E_s/N_0}}\right) - (1+\rho)\cdot\left(\frac{1}{2\rho^2 D}+ \frac{(2(1+(1/(\rho D)))/(\rho^2 D))+(4/(\rho^2 D E_s/N_0))}{4((1+(1/(\rho D)))^2+(4/(\rho D E_s/N_0)))^{\frac{1}{2}}}\right).$$

Equating this function to zero allows the result to be simplified into a quadratic equation in ρ:

$$D\left[\frac{(D-1)E_s}{N_0}-1\right]\rho^2+2D\rho+\left[\frac{(1-D)E_s}{N_0}-D\right]=0.$$

The solutions for ρ_0 are then given by:

$$\rho_0 = \frac{D \pm \sqrt{(E_s/N_0)(1+(E_s/N_0))D(D-1)^2}}{D(1-(E_s/N_0)(D-1))}. \quad (8)$$

Numerical experience indicates that usually only one of these solutions is a valid solution to $\partial F/\partial\rho = 0$. This outcome arises because a squaring operation is used to remove the square root terms in $\partial F/\partial \rho$ when simplifying to the quadratic equation. It is the squaring operation which appears to introduce an extra, possibly incorrect solution for ρ_0. The correct solution may be verified as follows: (1) ignoring negative values of ρ_0, (2) checking that $\partial F/\partial\rho = 0$ or (3) picking the value of ρ_0 which has the smaller value of $F(\rho_0)$. In the results obtained for this paper, it was found that substituting "$-$" for "$\pm$" into (8) always gave the correct solution, though it has not been proven that this is the case for all possible scenarios.

For large values of E_s/N_0, the solution for ρ_0 simplifies to:

$$\rho_0 = \sqrt{\frac{1}{D}}. \quad (9)$$

Substituting this back into equation (7) for $F(\rho_0)$ gives:

$$F(\rho_0) \approx (1+\rho_0)\left(1+\frac{1}{\rho_0 D}\right) = \left(1+\sqrt{\frac{1}{D}}\right)^2. \quad (10)$$

It is apparent that for high SNRs, the larger the value of D, the smaller is the value of ρ and of the resulting performance loss due to channel estimation. As might be expected, the use of pilot signals is more efficient for low Doppler rate channels than for very fast fading environments. In the extreme case where $D = 1$, ρ_0 always becomes 1 which is consistent with [8].

4 MULTIPLE TRANSMIT ANTENNAS CASE

This section will now analyse the general case of an (n, m) system employing a space-time code, using results from [2, 9]. Initially, two types of space-time code which have been used for simulation work are introduced. Then, a result for WEL will be derived, followed by analysis of optimising the pilot-data symbol ratio ρ.

4.1 SPACE-TIME TRELLIS CODES

This paper will examine WEL effects for two different types of space-time codes, namely space-time trellis codes and space-time block codes. The specific codes that have been considered are now described.

1. MC-STTC-256: In this paper a 256 state space-time trellis code called MC-STTC-256 will be considered. This was first presented in [10] and consists of a single rate 1/2, constraint length 9 convolutional encoder with generator polynomials octal 561 and 753, whose outputs are mapped in pairs in a serial-to-parallel fashion to n QPSK modulators. The n outputs are transmitted by the n transmit antennas. This code has not been proven to be an optimum STTC, according to the criteria of [2]. However, the large constraint length means the decoder should be able to exploit most of the available diversity in the MIMO system.

Since the transmitter is using a trellis code, it may be decoded with a Viterbi decoder, which calculates the following metric for a given codeword of length l_0 symbols:

$$\mathcal{M}(e_l^i, l=1\ldots l_0, i=1\ldots n) = \sum_{l=1}^{l_0}\sum_{j=1}^{m}\left| r_l^j - \frac{\mu\sqrt{E_s}}{\sigma}\sum_{i=1}^{n} e_l^i \beta_{i,j}\right|^2, \quad (11)$$

with parameters $\sigma = \sqrt{1+\sigma_e^2}$ and $\mu = \sigma^{-1}$. The quantities $\{e_l^i, l = 1\ldots l_0, i = 1\ldots n\}$ are the M-PSK complex constellation points corresponding to the given path.

The MC-STTC-256 code uses QPSK modulation, so that $M = 4$ in this case. This metric may be recalculated for all possible trellis paths, in order for the Viterbi algorithm to select the most likely one.

2. Space-Time Block Codes (STBC): Space-time block codes are a special class of space-time trellis codes [9]. These codes are designed to maximise the order of diversity achieved by receiver decoding, but do not provide additional coding gain. However, the transmitted symbols are encoded using an orthogonal structure which simplifies receiver processing. This paper will focus on two STBCs that have been proposed for use in CDMA systems, under the name of space-time spreading (STS) [11]. Both codes can only be used for real transmitted signal constellations, such as BPSK.

The first code to be considered is a rate 1/2 code for the case of two transmit antennas. The code starts with two BPSK modulated symbols b_1 and b_2, which are to be communicated to the receiver. These are mapped to the transmission symbols c_l^i as follows:

$$\begin{bmatrix} c_1^1 & c_2^1 \\ c_1^2 & c_2^2 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 \\ b_2 & -b_1 \end{bmatrix}. \quad (12)$$

Equation (12) shows that this code achieves 1 bit/s/Hz, since two BPSK symbols are sent in two symbol intervals. The sampled signals for symbols $l = 1$ and 2 at antenna j are:

$$\begin{bmatrix} r_1^j \\ r_2^j \end{bmatrix} = \begin{bmatrix} \alpha_{1,j} & \alpha_{2,j} \\ -\alpha_{2,j} & \alpha_{1,j} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} \eta_1^j \\ \eta_2^j \end{bmatrix}.$$

The BPSK symbols b_1 and b_2 could be obtained using the Viterbi algorithm with the metric of equation (11). However, a simpler, but still optimum, decoding procedure is obtained from the orthogonal structure of the transmitter matrix in equation (12):

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \end{bmatrix} = \mathrm{sgn}\left(\Re\left\{\sum_{j=1}^{m} \begin{bmatrix} \beta_{1,j}^* & -\beta_{2,j}^* \\ \beta_{2,j}^* & \beta_{1,j}^* \end{bmatrix} \begin{bmatrix} r_1^j \\ r_2^j \end{bmatrix}\right\}\right), \quad (13)$$

where * denotes the complex conjugate operation. The notation $\Re\{\ \}$ denotes the real part and sgn( ) the sign. In equation (13), the exact channel coefficients $\alpha_{i,j}$ have had to be replaced by the receiver's estimates $\beta_{i,j}$.

The second code is a rate 1/4 code for the case of four transmit antennas. The code maps four BPSK symbols b_1 to b_4 into transmission symbols as follows:

$$\begin{bmatrix} c_1^1 & c_2^1 & c_3^1 & c_4^1 \\ c_1^2 & c_2^2 & c_3^2 & c_4^2 \\ c_1^3 & c_2^3 & c_3^3 & c_4^3 \\ c_1^4 & c_2^4 & c_3^4 & c_4^4 \end{bmatrix} = \begin{bmatrix} b_1 & -b_2 & -b_3 & -b_4 \\ b_2 & b_1 & b_4 & -b_3 \\ b_3 & -b_4 & b_1 & b_2 \\ b_4 & b_3 & -b_2 & b_1 \end{bmatrix}. \quad (14)$$

The 4×4 matrix in equation (14) is again an orthogonal design. This means that the four bits can be linearly separated at the receiver in an analogous fashion to equation (13). As with equation (12), this code achieves 1 bit/s/Hz.

Now that the channel and receiver models have been defined, the next section will consider the weight error loss for multiple transmit antenna systems employing space-time trellis coding.

4.2 WEL FORMULA

An upper bound on the pairwise error probability (PEP) for choosing a space-time code vector $\mathbf{e} = [e_1^1, e_1^2, \ldots e_{l_0}^n]$ in place of the correct vector $\mathbf{c} = [c_1^1, c_1^2, \ldots c_{l_0}^n]$ has been derived for space-time trellis coding with imperfect channel estimation in [9]. The result for a Rayleigh fading channel with any equal energy constellation (e.g. M-PSK) is given in equation (12) of [9]:

$$P_p(\mathbf{c} \to \mathbf{e}, \mu, E_s/N_0) \leq \left(\frac{1}{\prod_{i=1}^{n}\left(1 + \lambda_i(E_s/4N_0)\left(\frac{|\mu|^2}{1+n(1-|\mu|^2)(E_s/N_0)}\right)\right)}\right)^m \quad (15)$$

The parameters λ_i are independent of SNR and are eigenvalues of a matrix related to $\mathbf{c}$ and $\mathbf{e}$ – see [2] for full details on the design and performance of space-time trellis codes.

The procedure for deriving the WEL is the same as that used in section 3. The PEP for ideal channel estimation $P_p(\mathbf{c} \to \mathbf{e}, 1, E_s/N_0)$ is equated with $P_p(\mathbf{c} \to \mathbf{e}, \mu, \mathrm{WEL} \cdot E_s/N_0)$, the PEP with non-ideal channel estimation to give:

$$\begin{aligned} \frac{E_s}{4N_0} &= \frac{\mathrm{WEL} \cdot E_s|\mu|^2}{4N_0(1 + n(1-|\mu|^2)(\mathrm{WEL} \cdot E_s/N_0))} \\ \Rightarrow 1 &= \frac{\mathrm{WEL}}{1 + (n/L) + (1/(\mathrm{WEL} \cdot LE_s/N_0))}. \end{aligned}$$

Solving for WEL gives:

$$\mathrm{WEL} = \frac{1}{2}(1 + (n/L)) \pm \left(\frac{\sqrt{(1 + (n/L))^2 + (4/(L \cdot E_s/N_0))}}{2}\right). \quad (16)$$

As with equation (4), replacing $\pm$ by $+$ and using the substitution $E_b = mE_s/b$ yields:

$$\mathrm{WEL} = \left(1 + \frac{n}{L}\right) + \left(\sqrt{\left(\frac{1+(n/L)}{2}\right)^2 + \frac{m}{bL \cdot E_b/N_0}} - \frac{(1+(n/L))}{2}\right). \quad (17)$$

An upper bound on the probability of bit error for the given space-time trellis code may be obtained by summing (and scaling) the values of $P_p(\mathbf{c} \to \mathbf{e})$ for all possible error codewords $\mathbf{e}$ [1]. The WEL is the same for any pair of codewords $\mathbf{c}$ and $\mathbf{e}$. This is because changing $\mathbf{c}$ or $\mathbf{e}$ only alters the eigenvalues $\{\lambda_i\}$ and it has already been noted that they are independent of SNR. Therefore, the WEL formula applies unchanged to the upper bound on probability of bit error for the space-time trellis code.

The "high SNR" WEL formula is equal to $1 + (n/L)$, rather than $1 + L^{-1}$ as for the single transmit antenna case. This indicates that as n increases, the value of ρ (and hence $L = \rho D$) must increase by n to avoid any increase in WEL due to channel estimation errors. The reason for this is that each state of the Viterbi metric in (11) is comprised of the sum of n channel estimates, rather than just 1. The error variances of the n individual channel estimates add, so that the error variance for n transmit antennas is n times larger than the single transmit antenna case. Since there are now n transmit antennas, the total transmit power allocated to pilot symbols is proportional to ρn. Therefore, the total pilot power allocation at the transmitter must increase by n^2 to keep the WEL at the same level as for the single antenna case. If the code rate and modulation scheme are unchanged, the corresponding data throughput of the system can only increase linearly with n. Setting $n = 1$ in (17) yields the same formula as the exact result in (5). The BER of space-time block codes with channel estimation errors has been analysed on page 56 of [11] for the case of n transmit antennas and $m = 1$ receive antenna. Equation (18) could also be obtained from the results presented in [11].

4.3 Optimising The Pilot-Data Symbol Ratio

The analysis of optimising the pilot-data symbol ratio ρ proceeds in the same manner as for the single transmit antenna case. The WEL and pilot overhead function $F(\rho)$ can be defined for a single transmit antenna as:

$$F(\rho) = \mathrm{WEL} \cdot (1+\rho) = \frac{(1+\rho)}{2}\left(1 + \frac{n}{\rho D} + \sqrt{\left(1+\frac{n}{\rho D}\right)^2 + \frac{4}{\rho D E_s/N_0}}\right). \quad (18)$$

As with equation (7), differentiating F and setting it equal to 0 yields:

$$D\left[\frac{E_s}{N_0}(D-n) - 1\right]\rho^2 + 2D\rho + \left[\frac{nE_s}{N_0}(n-D) - D\right] = 0.$$

The optimum value of ρ, ρ_0, is then:

$$\rho_0 = \frac{D \pm \sqrt{(E_s/N_0)(1+(nE_s/N_0))D(D-n)^2}}{D(1-(E_s/N_0)(D-n))}. \quad (19)$$

As with equation (8), only one solution will be valid. For large values of E_s/N_0, ρ_0 simplifies to $\sqrt{n/D}$, which gives a combined overhead of:

$$F(\rho_0) \approx \left(1 + \frac{n}{\rho_0 D}\right)(1+\rho_0) = \left(1 + \sqrt{\frac{n}{D}}\right)^2. \quad (20)$$

As with the single transmit antenna case, increasing D reduces the combined overhead due to channel estimation and WEL. It is interesting to note that while the WEL in (17) is proportional to n, the corresponding value of ρ_0 is only proportional to $\sqrt{n}$. This implies that the unavoidable performance loss $F(\rho_0)$ will increase with n, which is evident from the high SNR result in (20).

5 RESULTS AND EXAMPLES

This section will present example results and simulations to confirm the theoretical equations and illustrate some of the points made about the expressions that were obtained. To begin with, the single transmit antenna case is considered.

5.1 Single Transmit Antenna Results

Some simulations are now presented to illustrate the WEL expression in equation (4) and the optimum value ρ_0 given in equation (8). An $m = 2$ antenna diversity receiver is considered to provide an example of how ρ may be optimised. For this case, ideal coherent, uncoded BPSK requires an E_b/N_0 value of 8.45 dB for a mean BER of 1% [1]. The E_b/N_0 value (in linear scale) is then multiplied by the function $F(\rho)$ in equation (7) to yield the equivalent E_b/N_0 result *including* WEL and pilot overhead (PO). The task is then to find the optimum value of ρ_0 corresponding to the minima of these curves. Table 1 presents results for ρ_0 and E_b/N_0, including WEL and PO for three different values of D. Figure 3 then plots the ideal BPSK result along with the pilot signal system results for E_b/N_0, showing the optimum values ρ_0. It can also be seen that when the channel fading is rapid, and hence the number of data symbols D is small, the unavoidable performance loss becomes higher.

Table 1: Table of results for the optimum pilot-data symbol ratio ρ_0 and required E_b/N_0 (including WEL and pilot overhead), for a scenario with $m = 2$ antennas and a BER target of 1%.

D	ρ_0	E_b/N_0
25	0.217	10.19 dB
100	0.111	9.38 dB
400	0.0560	8.92 dB

5.2 Multiple Transmit Antenna Results

This part of the paper will compare the WEL formula in equation (17) with results for for the MC-STTC-256 and STS codes. Results for required E_s/N_0 to achieve a 10%

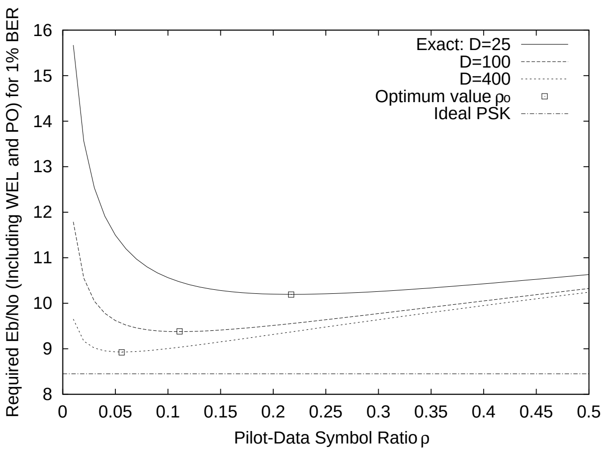

Figure 3: Plot of the total required E_b/N_0 value (including WEL and pilot overhead (PO)) for a BER of 1% vs the pilot-data symbol ratio ρ and the number of data symbols D, with $m = 2$ receive antennas. The optimum pilot-data symbol ratio ρ_0 from Table 1 is also plotted for each curve.

frame error rate (FER) for MC-STTC-256 are presented in column 2, Table 1 of [10] for various array sizes n and m, assuming a frame size of 400 binary bits. The simulations reported here consider a frame size of 192 bits (including an 8 bit encoder tail of zeros). It was found that halving the frame size in bits reduced the required E_s/N_0 by approximately 1 dB in all cases relative to the results reported in [10]. The exact results for E_s/N_0 for the 192 bit frame are shown in table 2.

Table 2: E_s/N_0 results for 10% FER using a frame size of 192 bits and different (n,m) configurations for the MC-STTC-256 code.

Array Sizes (n, m)	E_s/N_0 for 10% FER
(1,1)	10.2
(2,1)	8.6
(2,2)	2.4
(4,1)	10.7
(4,2)	3.0
(4,4)	-2.3

Table 3 presents a comparison of WEL results obtained from equation (16) and from direct simulation of the MC-STTC-256 receiver. The simulations are reported in terms of a channel estimation parameter L', which represents the *total* number of pilot symbols devoted to training within the frame. Therefore, the number of pilot symbols at each antenna is obtained as $L = L'/n$. In the case of the theoretical results, the E_s/N_0 results from table 2 were used in the WEL calculation, along with the correct value of L for each array size n. Columns 3 and 4 of table 3 show a remarkable similarity, at least in the order of WEL result that is obtained in both cases. It should be emphasised that the WEL formula of equation (17) is based on a PEP approximation. However, both sets of results show an approximately n^2 increase in WEL for this simulation. This arises from two factors: (1) the WEL formula in (17) is proportional to n and (2) the use of a constant number of pilot symbols L' means that the L is inversely proportional to n for these results. The combined effect is that the WEL results for this simulation increase proportionally with n^2 because the total number of pilot symbols L' is fixed for all array sizes.

Table 3: Comparison of WEL results obtained using the WEL equation (17) and those from simulation. The notation (n, m) denotes the number of transmit and receive antennas respectively. The channel estimation parameter $L' = L/n$.

		WEL for 10% FER (dB)	
(n,m)	L'	Theory	Simul
(1,1)	1	3.11	2.8
	10	0.45	0.4
	100	0.047	$<$ 0.1
(2,1)	1	7.04	7
	10	1.52	1.5
	100	0.18	$<$ 0.1
(2,2)	1	7.18	7.5
	10	1.70	2
	100	0.22	0.2
(4,1)	1	12.31	$>$ 9
	10	4.17	4.5
	100	0.66	0.6
(4,2)	1	12.33	$>$ 11
	10	4.27	4.5
	100	0.71	0.9
(4,4)	1	12.40	12.4
	10	4.53	4.6
	100	0.85	0.8

Finally, parts (a) and (b) of Figure 4 present WEL results for the STBC codes presented in equations (12) and (14) respectively. In all cases, the receiver is attempting to attain a BER of 0.01. The results show a good match between theory and simulation in both cases. They also show that the WEL tends to increase as the receiver's array size m increases, because the E_s/N_0 value at each receive antenna is reduced.

6 DISCUSSION AND CONCLUSIONS

This paper has investigated the effects of non-ideal channel estimation on coherent communications systems using multiple antennas both at the transmitter and receiver.

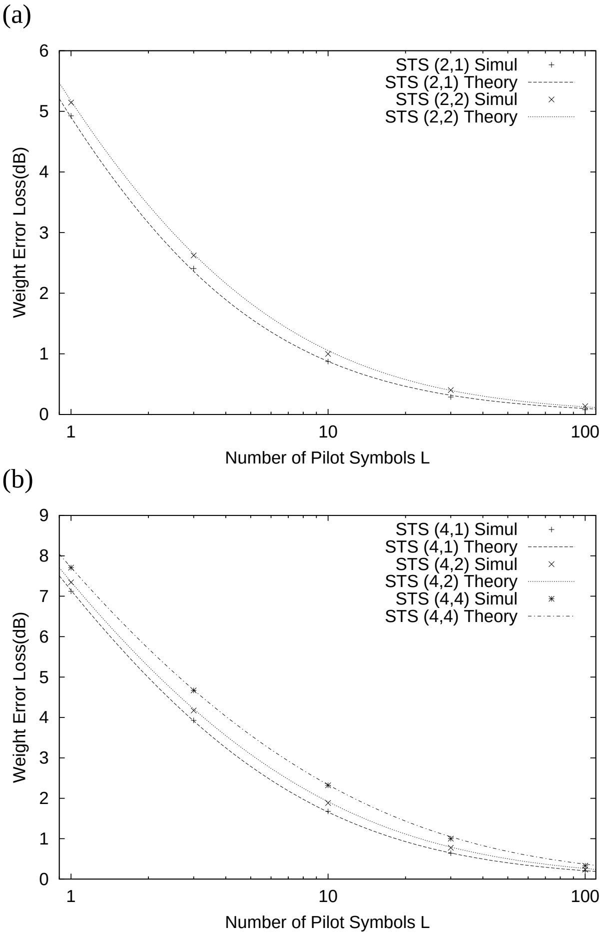

Figure 4: Plot of theoretical (lines) and simulated (points) results for WEL vs number of pilot symbols L for a BER of 0.01. The code used was (a) $n = 2$ transmit antenna STS and (b) $n = 4$ transmit antenna STS.

Exact results have been derived for weight error loss and the optimum pilot-data symbol ratio ρ_0 for an uncoded M-PSK system operating in a Rayleigh fading environment, which employs only multiple antennas at the receiver. Results have also been derived for the case where multiple antennas are used at both transmitter and receiver in a Rayleigh fading environment, along with an M-PSK based space-time trellis code. High SNR results show that as the transmitter array size n is increased, the number of pilot symbols L at each antenna must be multiplied by approximately n to prevent the weight error loss increasing. This leads to an increase of order $O(n^2)$ in the total power allocated to pilot symbols at the transmitter, while the overall system data rate is likely to increase only by order $O(n)$. The results for minimising the combined WEL and pilot overhead only increase ρ_0 by a factor of $\sqrt{n}$ rather than n. This suggests that accurate channel estimates are easier to obtain in slow fading environments and are thus of particular interest for space-time trellis code applications. Finally, simulation results in section 5 of the paper back up the trends observed in the theoretical analysis of these systems.

It would be interesting to investigate whether the trends observed for space-time trellis coding also apply to other MIMO techniques. An investigation of the BLAST scheme in [12] indicated that the number of pilot symbols should also increase linearly with the number of transmit antennas n.

ACKNOWLEDGEMENTS

John Thompson gratefully acknowledges funding by the UK EPSRC and Nortel Networks. He also thanks Tony Koutalos for his help with space-time spreading codes.

Manuscript received on September 13, 2001.

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