BEAM FALSING LOSS IN SWITCHED-BEAM SMART ANTENNA SYSTEMS

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Abstract - Expressions for the 'Beam Falsing Loss' (BFL) are derived for the uplink of a TDMA land mobile radio system using a switched-beam Base Station architecture. The analysis shows, for representative system parameters and channel assumptions, that in terms of mean BER, the performance degradation owing to the beam falsing is small.

I. INTRODUCTION

Multibeam receiver architectures ('smart antennas') have been demonstrated to offer significant benefits such as coverage and capacity improvement for cellular land mobile radio systems [1]. In this paper we analyse the performance of a fixed-beam ('switched-beam') receiver architecture in the context of the uplink of a TDMA radio system, when beam selection is not ideal. We present performance in terms of 'Beam Falsing Loss' (BFL, measured in dB) compared to a 'perfect' beam selection processor for the case of a mobile located in the beam peak and with low angular spread of the signal impinging on the Base Station (BS) antenna array. We discuss how historical information could be used to reduce this BFL if required, and discuss the implications of the mobile being located away from peak-of-beam, in terms of BFL and additional so-called 'cusping loss'. Finally, we go on to make some comparisons with an alternative 'adaptive-beam' architecture, and note that the comparison is affected by considerations such as cusping-loss (from which adaptive-beam systems don't suffer) and signal angular spread.

II. SYSTEM DESCRIPTION

For ease of analysis we consider a system using coherent BPSK or QPSK modems, and a timeslotted 'burst' structure with an in-built training sequence (TS), similar to IS136 [2] and GSM [3]. This burst structure is represented diagramatically in Fig. 1. We assume that the received signals are Rayleigh faded, with a doppler characteristic such that the fading is approximately constant over the duration of a burst, but is independent from burst-to-burst.

Assuming a flat (dispersionless) channel and a narrow angular spread of the received signal, the fast (Rayleigh) fading as seen at the antenna elements will be totally correlated from element to element. For ease of implementation the beams are orthogonally generated using a passive beamformer processor (BFP) structure such as a Butler matrix. We assume that the mobile is located in a beam peak, as shown in Fig. 2.

The BS architecture is such that there is a separate receiver connected to each of the beam outputs, feeding a separate correlator for correlation of the received TS (where perfect sample timing for this correlation process is assumed). The 'best beam' for demodulation of the given burst is decided by comparing the detected energies at the beam correlator outputs. If the 'wrong' beam is erroneously selected for demodulation of a burst, then we term this a 'beam false' event.

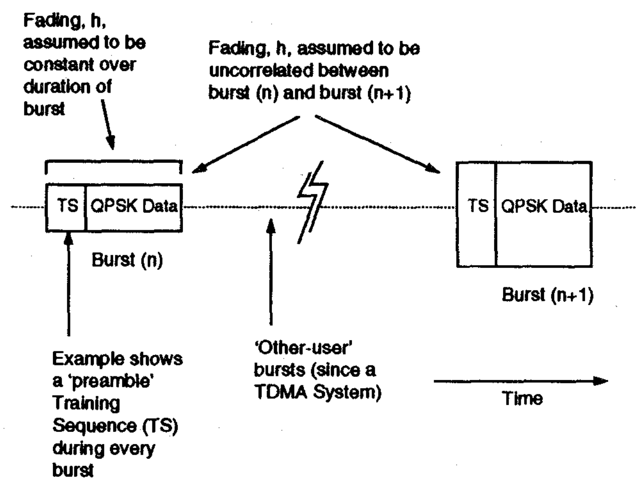

Fig. 1 Assumed TDMA Signal Format

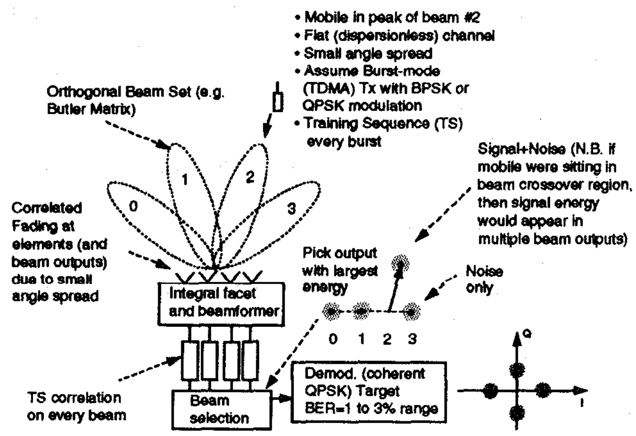

Fig. 2 Typical Switched-Beam Configuration

III. ANALYSIS

A. Preliminaries

The analysis for the beam falsing rates is analogous to that for the determination of symbol error rate for incoherent demodulation of M-ary orthogonal modulation [4, p164 (4.129)] if the TS is considered to be a 'symbol', and the wrong beams contain noise only. Thus we can write that the probability of a beam falsing event for an M-beam system in a static (non-fading) channel, p_{MS}, is given by[1]

$$p_{MS}(\mu_b, L_S, M) = \frac{e^{-\mu_b L_S}}{M}\sum_{n=2}^{M}(-1)^n\binom{M}{n}e^{\frac{+\mu_b L_S}{n}} \qquad (1)$$

where

The subscript 'MS' in p_{MS} signifies a beam falsing probability (BFP) for an M-beam system in the *static* channel.

μ_b equals *mean Eb/No*, the ratio of bit energy to noise power spectral density (at the output of the wanted beam), which for the static channel also equals the *instantaneous Eb/No*

L_S is the number of bits in the TS

M is the number of beams

In order to calculate the beam falsing probability for the Rayleigh fading channel, p_{MR}, we must integrate the static channel BFP of (1) over the probability density function (PDF) of the instantaneous *Eb/No*, $p_x(x)$, given by [5, p.29, (1.1.123)]

$$p_x(x) = \frac{1}{\mu_b}e^{-\left(\frac{x}{\mu_b}\right)} \qquad (2)$$

where μ_b here represents the *mean* received *Eb/No*, and x represents the instantaneous *Eb/No*. This integral is represented as follows:

$$p_{MR}(\mu_b, L_S, M) = \int_{x=0}^{\infty}\frac{1}{\mu_b}e^{-\left(\frac{x}{\mu_b}\right)}\frac{e^{-L_S x}}{M}\sum_{n=2}^{M}\left[(-1)^n\binom{M}{n}e^{\frac{-L_S x}{n}}\right]dx \qquad (3)$$

which can be evaluated without difficulty (after reversing the order of summation and integration) to give:

$$p_{MR} = \frac{1}{\mu_b M}\sum_{n=2}^{M}\frac{(-1)^n}{\alpha}\binom{M}{n} \qquad (4)$$

where

$$\alpha = \left(L_S - \frac{L_S}{n} + \frac{1}{\mu_b}\right) \qquad (5)$$

Now given our assumption of BPSK or QPSK modulation, our coherent demodulator instantaneous bit error rate (BER) *given the assumption of perfect beam selection* is given by [5, p246, (4.2.40)]

$$p_{BS} = \int_{x=\sqrt{2\mu_b}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{x^2}{2}\right)}dx = Q(\sqrt{2\mu_b}) \qquad (6)$$

where the subscript 'BS' in p_{BS} signifies a BPSK/QPSK bit-error probability for the *static* channel.

By integrating the BER characteristic (6) over the Rayleigh fading (2) we obtain the standard result for the BER [5, p.717, (7.3.7)] as follows

$$p_{BR} = \frac{1}{2}\left(1 - \sqrt{\frac{1}{1 + 1/\mu_b}}\right) \qquad (7)$$

where the subscript 'BR' in p_{BR} signifies a BPSK/QPSK bit-error probability for the *Rayleigh fading* channel (and ideal beam selection). At high signal-to-noise ratio (SNR), it has been shown [5, p.719, (7.3.13)] and [6] that (7) can be approximated as follows

$$p_{BR} \approx \frac{1}{4\mu_b} \qquad (8)$$

B. BER Increase due to Beam Falsing

Now we investigate the receiver BER in the static channel when we combine the effects of beam falsing and bit errors. As shown in Fig. 1, the data is formatted into separate 'bursts', where each burst is assumed to have constant amplitude over its duration, but there is independent Rayleigh fading from burst to burst. If the beam is correctly selected (with probability $1\text{-}p_{MS}$), then the bit error probability for that burst is as given in (6). However since the mobile is located in one of the beam peaks (Fig. 2), and we employ an orthogonal beam set, then if the beam is *falsely* selected, zero received signal power is passed to the demodulator, and the probability of bit error on the burst will be 0.5. Thus it can be seen that the *overall* bit error probability, averaged over the 'good' and 'bad' bursts is given by:

$$\begin{aligned} p_{BSF} &= (1 - p_{MS})\cdot p_{BS} + \frac{1}{2}\cdot p_{MS} \\ &= p_{BS} + \frac{1}{2}\cdot p_{MS} - p_{BS}\cdot p_{MS} \end{aligned} \qquad (9)$$

where the subscript 'BSF' in p_{BSF} denotes bit error probability for the static channel where we take into account the effect of beam falsing. Thus the 'delta increase' in BER in the static channel due to the effect of beam falsing, denoted Δp_{BSF}, is given by

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[1] Note that the variables within brackets after $p_{MS}(..)$ here are the input parameters. In many cases these are not explicitly stated in full in the following work, for notational convenience, and due to space constraints.

$$\Delta p_{BSF} = p_{BSF} - p_{BS} = \frac{1}{2}\cdot p_{MS} - p_{BS}\cdot p_{MS} \qquad (10)$$

To obtain the BER with beam falsing in a Rayleigh fading channel, p_{BRF}, we must integrate the static channel BER of (9) over the Rayleigh fading PDF of (2) as follows:

$$p_{BRF}(\mu_b) = \int_{x=0}^{\infty} \frac{e^{-\left(\frac{x}{\mu_b}\right)}}{\mu_b}\left[p_{BS}(x) + \frac{1}{2}\cdot p_{MS}(x) - p_{BS}(x)\cdot p_{MS}(x)\right]dx \qquad (11)$$

The integral of (11) can be split into three separate integrals, two of which have already been evaluated in (4)(7). Thus we can write

$$\begin{aligned} p_{BRF}(\mu_b) &= p_{BR}(\mu_b) + \left[\frac{1}{2}p_{MR}(\mu_b) - p_{INT}(\mu_b)\right] \\ &= p_{BR}(\mu_b) + \Delta p_{BR}(\mu_b) \end{aligned} \qquad (12)$$

where $p_{INT}(\mu_b)$ represents the third term of the integral in (11). $\Delta p_{BR}(\mu_b)$ represents the 'delta increase' in BER in Rayleigh fading due to the beam falsing, compared to the 'ideal' case of (7).

We now derive the probability $p_{INT}(\mu_b)$. By reference to (11)(12)

$$p_{INT}(\mu_b) = \int_{x=0}^{\infty} \frac{e^{-\left(\frac{x}{\mu_b}\right)}}{\mu_b}\left[p_{BS}(x)\cdot p_{MS}(x, L_S, M)\right]dx \qquad (13)$$

Substituting in the values of $p_{MS}(x)$ and $p_{BS}(x)$ from (1)(6) respectively we obtain the expression

$$p_{INT}(\mu_b) = \int_{x=0}^{\infty} \frac{e^{-\left(\frac{x}{\mu_b}\right)}}{\mu_b} Q(\sqrt{2x})\frac{e^{-L_S x}}{M}\sum_{n=2}^{M}\left[(-1)^n\binom{M}{n}e^{\frac{-L_S x}{n}}\right]dx \qquad (14)$$

Equation (14) can be simplified by reversing the order of summation and integration. We also expand the expression for Q(√(2x)) from (6), and use the value of α from (5) as follows

$$\begin{aligned} p_{INT}(\mu_b) &= \frac{1}{\mu_b\cdot M}\sum_{n=2}^{M}(-1)^n\binom{M}{n}\int_{x=0}^{\infty} e^{-\alpha\cdot x}\int_{y=\sqrt{2\cdot x}}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}dy\,dx \qquad (15) \\ &= \frac{1}{\mu_b\cdot M}\sum_{n=2}^{M}(-1)^n\binom{M}{n}\cdot D(\alpha) \end{aligned}$$

where

$$D(\alpha) = \int_{x=0}^{\infty} e^{-\alpha\cdot x}\int_{y=\sqrt{2\cdot x}}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}dy\,dx \qquad (16)$$

So in order to find the value of $p_{INT}(\mu_b)$ in (12)(15) we must solve for $D(\alpha)$ in (16). To do this we make use of the identity

$$\int_{x=0}^{\infty} f_x(x)\left[\int_{y=g(x)}^{\infty} f_y(y)dy\right]dx = \int_{y=0}^{\infty} f_y(y)\left[\int_{x=0}^{g^{-1}(y)} f_x(x)dx\right]dy \qquad (17)$$

which is a process of reversing the order of integration within a double-integral expression. Thus we can rearrange (16) as follows

$$D(\alpha) = \int_{x=0}^{\infty} e^{-\alpha\cdot x}\int_{y=\sqrt{2\cdot x}}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}dy\,dx$$

$$D(\alpha) = \int_{y=0}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}\int_{x=0}^{y^2/2} e^{-\alpha\cdot x}dx\,dy$$

$$D(\alpha) = \frac{-1}{\alpha}\int_{y=0}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}\left[e^{-\left(\frac{\alpha y^2}{2}\right)} - 1\right]dy$$

$$D(\alpha) = \frac{-1}{\alpha}\left[\int_{y=0}^{\infty}\frac{e^{-\left[\frac{(1+\alpha)y^2}{2}\right]}}{\sqrt{2\pi}}dy - \int_{y=0}^{\infty}\frac{e^{-\left(\frac{y^2}{2}\right)}}{\sqrt{2\pi}}dy\right]$$

$$D(\alpha) = \frac{-1}{\alpha}\left[\sigma\int_{y=0}^{\infty}\frac{e^{-\left(\frac{y^2}{2\sigma^2}\right)}}{\sqrt{2\pi\sigma^2}}dy - \frac{1}{2}\right] \qquad (18)$$

where we have succeeded in rearranging the expression for $D(\alpha)$ into standard normal form using the identity

$$\sigma = \frac{1}{\sqrt{1+\alpha}} \qquad (19)$$

and so (18) simplifies considerably as follows

$$D(\alpha) = \frac{1-\sigma}{2\alpha} \qquad (20)$$

We substitute (20) back into (15), and the resulting expression for $p_{INT}(\mu_b)$, along with the expression for $p_{MR}(\mu_b)$ of (4), is substituted back into (12) to give the following simplified expression for the delta increase in the BER due to beam falsing in a Rayleigh fading channel.

$$\Delta p_{BR}(\mu_b, L_S, M) = \frac{1}{2\mu_b M}\sum_{n=2}^{M}(-1)^n\binom{M}{n}\left(\frac{1}{\alpha\sqrt{1+\alpha}}\right) \quad (21)$$

where the value of α is given by (5).

IV. RESULTS

In the previous section of this paper simplified expressions describing the increase in BER due to beam falsing have been derived, given the initial system assumptions of Fig. 1 and Fig. 2. How these equations behave for typical system parameters is now investigated. In this section we use similar arguments to those previously described (in more detail) in [6][2].

As in [6] we define F_{BR}, the *fractional increase* in BER, compared to an 'ideal' system (i.e. a system with zero beam falsing errors) as follows

$$F_{BR}(\mu_b, L_S, M) = \frac{\Delta p_{BR}(\mu_b, L_S, M)}{p_{BR}(\mu_b)} \quad (22)$$

where the values of $\Delta p_{BR}(\mu_b, L_S, M)$ and $p_{BR}(\mu_b)$ are given by (21) and (7) respectively. It has previously been shown that the BER characteristic $p_{BR}(\mu_b)$ at high SNR becomes linear (when SNR and BER are plotted on a log scale), since at high SNR we can use the approximation of (8). It is also apparent from consideration of (21) and (5) that at high SNR (and 'reasonably large' values of L_S) we can employ the approximation

$$\Delta p_{BR}(\mu_b, L_S, M) \approx \frac{1}{k\mu_b} \quad (23)$$

where 'k' is a constant determined by the values of L_S and M, since

$$\alpha = \left(L_S - \frac{L_S}{n} + \frac{1}{\mu_b}\right) \approx L_S\left(1 - \frac{1}{n}\right) \quad (24)$$

Thus the 'fractional increase in BER' given by (22) will tend to a constant value (of $4/k$) at high SNR (low BER). Due to this linear behaviour of the BER curve of (7) at high SNR, we can derive the beam falsing loss, which represents a 'reduction in receiver sensitivity' and, hence a 'horizontal displacement' of the BER curve, as follows [6]

$$BFL(dB) = 10\log_{10}\left(1 + F_{BR}\right) \quad (25)$$

Now since we have already argued that F_{BR} is itself constant at high SNR, then so is the BFL(dB) as given by (25), and thus the BFL is independent of the particular value of BER at which we choose to measure it, as long as we are within the region of high SNR (and thus low BER), where by 'high SNR' we mean that the approximation of (24) holds. Another way of saying this is that the dB reduction in receiver sensitivity due to the phenomenon of beam falsing is independent of the working BER, and it is purely determined by the number of beams, M, and the TS length L_S. Note however that many systems in practice will operate at a low SNR (high raw BER) in order to maximise capacity and coverage, whilst utilising coding to ensure acceptable transmission quality.

[2] although in that case the authors were considering errors in an adaptive beam, rather than a fixed beam configuration

To illustrate these points, taking the example of a 4-beam system as per Fig. 2, we plot the BER curve for the 'Magic Genie' (i.e. 'ideal') case, and curves including the effect of beam falsing in Fig. 3, for a number of different TS lengths. It can be seen that the BFL varies from around 0.9dB with an 8-bit TS, down to only 0.1dB for a 32-bit TS. Incidentally, there is excellent agreement between the curves of Fig. 3 which use (21), and those generated from numerical integration of the fundamental expressions (1)(6) over the Rayleigh fading (2) per (11) using the mathematical package MathCAD.

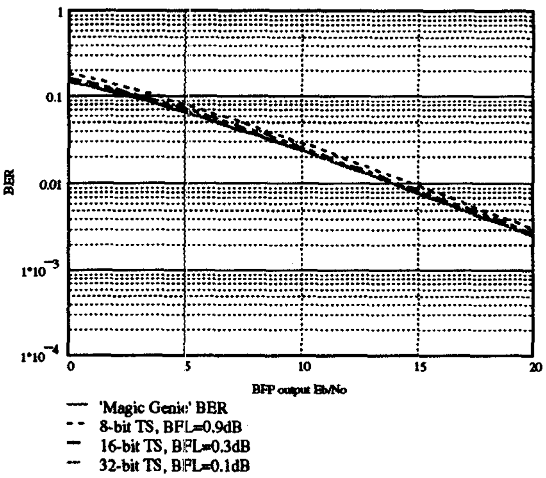

Fig. 3 Performance Curves Incorporating Beam Falsing Loss (M=4)

V. FURTHER CONSIDERATIONS

It can be seen from Fig. 3 that with a TS of appropriate length (e.g. 32 bits), then, for the representative scenario considered, the BFL can be reduced to very low levels (in this case a mere 0.1dB). In the analysis above, we make our decision about which beam to select for the current burst purely on the basis of the received signal power on said burst. We have assumed that the angle spread of the received signal is low. If we also assume that the mobile speed is low, compared to the burst rate, then the 'best' beam for the previous burst is also likely to be the 'best' beam for the current burst. Thus an alternative (or complementary) technique for achieving a low BFL could be to make use of information obtained from reception of previous bursts. By weighting our decision about the best beam for the current burst based on historical information (and there are a number of different ways this could be achieved), then we incorporate some 'prior knowledge' into our decision process, and hence can reduce our beam falsing probability.

The analysis within this paper has only considered the special case, whereby the wanted mobile is always located in a beam peak (see Fig. 2). In practice, it is possible that the mobile will be located somewhere in the 'cusping' or 'crossover' region where adjacent beams overlap. Such a situation is more difficult to analyse, since the beam selection problem is no longer analogous to a simple M-ary incoherent detection problem per (1), but rather is equivalent to an M-ary incoherent detection *with a frequency offset*. This is because if the mobile is located in the beam overlap region then energy will appear at the output of *all* of the beams (although *most* of the energy would, of course, appear in the outputs of the beams which straddle the mobile). Furthermore, if the 'wrong' beam is selected for a burst, then this doesn't automatically infer a 50% BER for that burst, since this 'wrong' beam may still contain some significant signal energy. The extreme case of course is the case where the mobile is received equally strongly in two adjacent beams, and either of them could equally be considered as the 'right' beam. Finally, even if the 'right' beam *is* correctly selected, then for a mobile which is not at beam peak there will be some loss, which we term 'cusping' loss, which is due to the 'scalloping' or 'roll-off' of the beam gain away from beam peak. For an orthogonal beam set this cusping loss will take a peak value of around 3.9dB, at the beam crossover point, and a mean value (averaged over all mobile positions) of some 1.2dB. In general, in order to analyse this case whereby the mobile can sit in the beam crossover regions it may be necessary to apply more approximate methods, or to resort to simulation.

In [7] a comparison is provided between the fixed-beam architecture described above (Fig. 2) and an alternative 'adaptive beam' architecture, which uses the same array of M antenna elements, but carries out maximal ratio combining (MRC) of the signals at these antenna elements (assuming, as above, a 'flat' or 'non-dispersive' channel). In this MRC case the analogue of beam-falsing loss is the 'weight error loss' (WEL), previously analysed in [6]. It is shown that in general the WEL is greater than BFL for a given TS length, but the advantage of the adaptive beam approach is that it doesn't suffer from cusping loss, and when the angular spread of the signal becomes significant it is much more effective at gathering all of the wanted signal energy, and also is able to provide some diversity gain.

VI. CONCLUSIONS

In this paper we have derived expressions for the BFL in a switched-beam system with narrow angular spread and a mobile located in the beam peak. The analysis shows, for representative system parameters, that the extra losses due the increase in mean BER owing to the beam falsing can be small. We conclude that further study is required to analyse the case when the mobile is located away from the beam peaks.

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